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Discussion paper

REPO RUNS

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Repo Runs

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Abstract

This paper develops a model of financial institutions that borrow short-term and invest into long-term marketable assets. Because these financial intermediaries perform maturity transformation, they are subject to runs. We endogenize the profits of the intermediary and derive distinct liquidity and solvency conditions that determine whether a run can be prevented. We first characterize these conditions for an isolated intermediary and then generalize them to the case where the intermediary can sell assets to prevent runs. The sale of assets can eliminate runs if the intermediary is solvent but illiquid. However, because of cash-in-the-market pricing, this becomes less likely the more intermediaries are facing problems. In the limit, in case of a general market run, no intermediary can sell assets to forestall a run, and our original solvency and liquidity constraints are again relevant for the stability of financial institutions.

Keywords: Investment banking, securities dealers, repurchase agreements, tri-party repo, runs, financial fragility.
JEL classification: E44, E58, G24

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1 Introduction

This paper develops a model of financial institutions funded by short-term borrowing and holding marketable assets. We show that such institutions are subject to the threat of runs similar to those faced by commercial banks and study the conditions under which runs can occur. We argue that profits are a key stabilizing element against runs, endogenize the profits, and derive distinct solvency and liquidity conditions for such institutions. Both conditions must hold for runs to be avoided. We also ask whether the sale of marketable assets can help prevent runs. If an institution is solvent, but illiquid, asset sales may help. However, as more institutions try to sell assets, their prices decline, limiting the amount that can be raised. In the limit, asset sales are completely ineffective. Indeed, if all borrowers try to sell assets, no institution is in a position to purchase them and the borrowers find themselves in the same situation as if their assets were not marketable.

Our framework is general and could be used to study several types of financial institutions that use short-term borrowing as a main source of financing. Such institutions include money market mutual funds, hedge funds, off-balance sheet vehicles including asset-backed commercial paper (ABCP) conduits, and structured investment vehicles (SIVs). We apply our model to large securities dealers who use the tri-party repo market as a main source of financing. This market is particularly interesting because of the key role it played during the Great Financial Crisis of 2007-09. It played a role in the collapse of Bear Stearns, which was triggered by a run of its creditors and customers, analogous to the run of depositors on a commercial bank.[2] This run was surprising, however, in that Bear Stearns's borrowing was largely secured – that is, its lenders held collateral to ensure repayment even if the company itself failed. However, given the illiquidity of markets in mid-March, creditors may have lost confidence that they could recoup their loans by selling the collateral. Many short-term lenders declined to renew their loans, driving Bear to the brink of default (Bernanke 2008). More generally, as noted by the Task Force on Tri-Party Repo Infrastructure (2009), "Tri-party repo

[2]See Duffie (2010) for more details on the dynamics that can lead to the failure of a dealer bank.

arrangements were at the center of the liquidity pressures faced by securities firms at the height of the financial crisis". The creation of the primary dealer credit facility (PDCF) provided a backstop for the tri-party repo market.

We develop a framework to study the fragility of dealers who hold marketable securities funded by short-term collateralized liabilities, building on the theory of commercial bank instability developed by Diamond and Dybvig (1983), Qi (1994), and others. In our view, there are important similarities between the fragility of commercial banking and securities trading. Our main goal is to exhibit and model these similarities, and to highlight the fundamental differences between securities dealers that borrow in the repo market against marketable securities as collateral and commercial banks that borrow unsecured deposits and hold nonmarketable loan portfolios.

A key contribution of our paper is to endogenize profits of dealers and show how profits are important to reduce financial fragility. Dealers have the choice between funding securities with their own cash or with short-term debt. We derive a dynamic participation constraint under which dealers will prefer to fund their operations with short-term debt and show that this condition implies that dealers make positive profits in equilibrium. These profits can be used to forestall a run and thus serve as a systemic buffer. If current profits are insufficient to forestall a run, dealers can boost current cash flows at the expense of future profits by distorting their investment strategy. We derive two constraints that can be interpreted as "solvency" and "liquidity" constraints and that are sufficient to prevent a run. The solvency constraint holds if the sum of current and future profits that can be mobilized through reductions in investment is sufficiently high to repay all investors. Even if the solvency constraint holds, however, the dealer must be able to have access to enough funds at the time of the run to prevent it. This occurs only if the liquidity constraint holds. Runs cannot occur if both constraints are satisfied.

While traditional banks hold opaque assets that are difficult to liquidate to meet withdrawals, securities dealers hold marketable assets that can potentially be sold to generate cash. We show that the ability to sell assets can help a dealer forestall a run if it is solvent but illiquid. How much asset sales can help, however, depends on various factors, including the market price of assets, that we identify in Section 6. Healthy dealers are willing to pay for assets up to the opportunity cost of their funds. As more assets are sold, the price of assets declines, and it becomes more difficult for a distressed dealer to raise cash. If several dealers simultaneously are in distress and attempt to sell their assets at the same time, cash-in-the market pricing (Allen and Gale 1994, Acharya and Yorulmazer 2008) limits this option further. In the limit, in the case of a market-wide run, no dealer is available to buy assets, and our liquidity and solvency conditions are necessary and sufficient to rule out market-wide runs.

Our theory is based on a dynamic rational expectations model with multiple equilibria. However, unlike in conventional models of multiple equilibria, not "everything goes" in our model. The theory pins down under what conditions individual institutions are subject to potential self-fulfilling runs, and when they are immune to such expectations. Since the intermediaries in our model are heterogenous and the liquidity and solvency conditions are specific to each institution, the theory makes predictions about individual institutions, and equilibrium is consistent with observations of some institutions failing and others surviving in case of changing market expectations.

An important economic function of the tri-party repo market, and of repo markets more generally, is to perform maturity transformation. An overnight repo is a short-term liability that is backed by a long-term asset, in the form of a security. Tri-party investors lend overnight repo and have access to their funds every morning, even if the securities that back the repos are not liquid. In "normal" times, maturity transformation is possible because there is a large number of tri-party lenders with largely independent needs for cash. On a given day, an individual lender may decide to "withdraw" its funds from the tri-party repo market by not rolling over the overnight loan. But in the aggregate, the amount of cash available in tri-party repos in our model will be stable by the law of large numbers. This is what happened in the market until 2007.

The maturity transformation provided by tri-party repo contracts resembles, in many ways, the maturity transformation achieved by commercial banks. Banks offer demand deposit contracts that allow the depositors to obtain their funds whenever they want. Yet, banks typically hold long-term assets. The decision of a depositor not to withdraw her funds from the bank is similar to the decision of a repo lender to reinvest. The bank can provide a demand deposit contract because it knows that depositors are unlikely to all withdraw their funds at the same time, but it is nevertheless vulnerable to coordination failures. We show that the same vulnerability can arise in other arrangements performing maturity transformation. In fact, the kind of strategic complementarities that can lead to runs in our model have also been found empirically in other types of intermediaries, notably mutual funds (see Chen, Goldstein, Jiang, 2010).

Conceptually, our theory of banking differs from that of Diamond and Dybvig (1983) in one important aspect. In Diamond and Dybvig (1983), deposit contracts are collective insurance devices for risk-averse households. In our framework, dealers interact with financial investors such as pension funds, money market funds and other institutions, for whom risk-aversion is probably not the right, and certainly not a robust assumption. We therefore do not place restrictions on investor preferences except for monotonicity. The *raison d'être* of banking in our model are fixed costs as in Diamond (1984). The creation, management, and marketing of securities is a specialized activity that requires the payment of fixed costs. Delegating these activities to a dealer is more efficient than having them performed by many small investors separately. Since this theory of delegation is standard, we do not develop it in this paper, and simply assume that the technology is only operated by dealers.

Our paper is complementary to Gorton and Metrick (2009), who point out the similarity between traditional bank runs and repo market instability. In particular, they argue that Repo rates, collateral, and other features of "securitized banking", as they call it, have counterparts in commercial banking. However, Gorton and Metrick (2009) do not propose a formal model of securitized banking and thus cannot identify the determinants of profits, liquidity, and solvency that are at the core of our analysis.[3] They document a large increase in haircuts for some repo transactions and argue that

[3] Shleifer and Vishny's "Unstable Banking" (2009) formalizes some elements of securitized banking, but focusses mostly on the spillover of irrational investor sentiments into the securitized loan market.

the rise in margins is akin to a run on the repo market. Their data does not include the tri-party repo market. Available data for the tri-party repo market, however, suggests that margins in the tri-party repo market did not increase much during the crisis, if at all. It appears that some tri-party repo investors prefer to stop financing a dealer rather than increase margins to protect themselves (see Task Force on Tri-Party Repo Infrastructure 2009). This is consistent with our model of expectations-driven runs and in contrast to the type of margin spirals described in Brunnermeier and Pederson (2009).

The remainder of the paper proceeds as follows: The next section provides an overview of the tri-party repo market. Section 3 describes our model. Section 4 characterizes steady states without runs. In particular, we derive the dealer's dynamic participation constraint in this section. Section 5 and 6 study the case of runs without and with asset sales, respectively. Section 7 discusses an extension of the model. Section 8 concludes.

2 An overview of the tri-party repo market

This sections provide an overview of the tri-party repo market, which is a lead example of our theory. We also describe below how our model could apply to other institutions such as money market mutual funds.

A repurchase agreement, or repo, is a sale of securities coupled with an agreement to repurchase the securities at a specified price on a later date (Garbade 2006). It resembles a collateralized loan, where the proceeds of the initial sale can be associated with the principal amount of the loan and the excess of the repurchase price over the sale price corresponds to the interest paid on the loan.

Tri-party repos are a popular form of repo contracts. As its name indicates, there are three parties to such a contract: the lender, the borrower, and a clearing bank. The lender is willing to lend cash against collateral. The borrower needs to obtain cash to fund its securities, which can be used as collateral. The clearing bank provides a variety of administrative services to support the transactions, including custody of securities, movement of cash and securities and valuation of collateral as well as optimization tools to support efficient collateral allocation. Because it deals with a large number of borrowers and lenders, the clearing bank can allocate the securities in the borrowers' portfolio to lenders willing to finance these securities in a very efficient way. In addition, the clearing bank can do this on its own books, thus avoiding the use and costs of a securities transfer and settlement system.

Lenders in the tri-party repo market typically want to be able to withdraw their funds on short notice. In particular, money market mutual funds and securities lending firms keep a substantial portion of their investments in overnight maturities to enable a quick response to sudden changes in client behaviors, including redemption requests. This is one reason that a large fraction of tri-party repo transactions are done on an overnight basis. Investing on an overnight basis can also be a way for the lender to control the amount of risk it takes. Indeed, the lender can decide not to roll over an overnight loan to a particular borrower if the risk attached to the borrower is perceived to increase.

Securities dealers are the main class of borrowers in the tri-party repo market. They seek to borrow funds to finance the securities they trade. Since dealers make money by buying and selling securities, it is very important for them to have the flexibility to easily substitute different types of securities that are used to collateralize their repos. In the U.S., this flexibility is most easily achieved in the tri-party repo market.

There are currently two clearing banks in tri-party repo market in the U.S.: JPMorgan-Chase (JPMC), and Bank of New York Mellon (BNYM). The clearing banks play a particularly important role for dealers by financing their securities during the day and allowing them to substitute collateral in a way that overnight investors may not agree to. As noted above, every afternoon the clearing banks allocates the borrowers' collateral to the lenders that are willing to finance it. The next morning, the clearing bank "unwinds" the previous night's repos by sending the cash back to the investors and the securities back to the dealers. Borrowers, however, would like to obtain financing for these securities until the next evening, when the new repo contracts are finalized. The clearing banks provide this intraday financing. In addition, the clearing banks allow the dealers to buy and sell the securities that serve as the collateral for the intraday part of the repo, as long as a sufficient amount of total collateral is available at all times. As mentioned, the ability to substitute collateral on an intra-day basis is particularly valuable to dealers.

The tri-party repo market reached a size of approximately $2.8 trillion in 2008, most of it overnight.[4] In this paper, we choose to focus on this market because it is an important source of funding for large securities dealers. Hence, a disruption in that market has severe consequences and could spill over to the broader financial markets.

While we focus on the case of the tri-party repo market, our theory is more general. For example, our model would apply to important classes of money market mutual funds. A popular class of money funds, the so-called 2a7 funds, offer a stable net asset value (NAV). This means that instead of having a fixed number of shares with a fluctuating price, the price of shares in these funds is fixed, typically at $1, and the number of shares changes. Since the number of shares cannot decrease, a fund with a NAV of less than $1 would have to be liquidated. This is called "breaking the buck". The stable NAV feature creates a sequential service constraint that makes these money funds particularly vulnerable to runs. In September 2008, the Reserve Primary Fund, which held Lehman commercial paper, broke the buck. This incident raised concerns about other money funds. During the week of September 15, investors redeemed about $300 billion from prime money market funds (ICI 2009). These events lead to the creation of a temporary guarantee program for money funds assets, which was announced on September 19, 2008, and expired a year later.

3 The Model

3.1 Framework

The economy lasts forever and does not have an initial date. It is populated by M infinitely-lived risk-neutral agents called dealers and indexed by $m \in \{1, ..., M\}$. Dealers are endowed with a very small amount of consumption goods, which we call cash, and have access to profitable investment

[4] Task Force on Tri-Party Repo Infrastructure (2009).

opportunities that we describe below.[5] Furthermore, at each date t, a continuum of mass N of "young" investors is born who live for three dates. Investors are born with an endowment of 1 unit of goods, that they can invest at date t and have no endowment thereafter. Investors' preferences for the timing of consumption are unknown when born at date t. At date $t+1$, investors learn their type. "Impatient" investors need cash at date $t+1$, while "patient" investors do not need cash until date $t+2$. The information about the investors' type and age is private, i.e. cannot be observed by the market. Ex ante, the probability of being impatient is α. We assume that the fraction of impatient agents in each generation is also α (the Law of Large Numbers).

The timing of the investors' needs of cash is uncertain because of "liquidity" shocks. In practice, repo investors, such as money market mutual funds, may learn about longer term investment opportunities and wish to redeploy their cash, or they may need to generate cash to satisfy sudden potential outflows from their own investors. We do not model explicitly what investors do with their cash in the event of a liquidity shock and, for the remainder of the paper, simply assume that they value them sufficiently highly to want to withdraw them from the repo market at the given point in time.[6] Their utility from getting repayments (r_1, r_2) over the two-period horizon can therefore simply be described by

$$U(r_1, r_2) = \begin{cases} u_1(r_1) & \text{with prob. } \alpha \\ u_2(r_2) & \text{with prob. } 1-\alpha \end{cases}$$

with u_1 and u_2 strictly increasing.[7]

[5] We do not model the clearing banks explicitly in this paper but think of them as being assimilated with the dealers. Hence, this model abstracts from the potential coordination problems between the dealers and the clearing banks.

[6] There are other ways of motivating the short-term nature of repos. Diamond and Rajan (2000, 2001) argue that short-term liabilities are a way to provide incentives to bankers who cannot commit to repay the proceeds of their investments to depositors. A similar argument can be made for dealers. Kashyap, Rajan, and Stein (2008) also emphasize the role of short-term liabilities to provide incentives.

[7] We do not assume the traditional consumption-smoothing motive of the Diamond-Dybvig literature, which would make little sense in our context.

Everybody in the economy has access to a one-period storage technology, which we think of as cash and that returns 1 for each unit invested. Dealers also have access to a long-term technology, which we think of as investment in, and possibly the creation of, securities. The long-term technology requires managerial expertise and other scarce resources and is therefore costly and subject to decreasing returns to scale. In terms of costs, a dealer m who wants to operate the long-term technology in a given period must pay a fixed operating cost $c_m \geq 0$ per period. Hence, dealers are potentially heterogenous in terms of their cost structure and therefore profitability. This has important empirical implications, which we explore in Section 7. For most of the analysis, however, we suppress the subscript m and consider a representative dealer with cost c.[8]

We model decreasing returns simply by assuming that there is a limit beyond which the long-term technology provides no returns. Hence, investing I_t units in securities at date t yields

$$\begin{cases} RI_t & \text{if } I_t \leq \overline{I} \\ R\overline{I} & \text{if } I_t \geq \overline{I} \end{cases} \tag{1}$$

with $R > 1$ at date $t+2$ and yields nothing at date $t+1$.[9] To simplify things, we assume that the return on securities is riskless.

The long-term investment returns given by (1) can only be realized by the dealer who has invested in the asset, because, as we discuss in the next subsection, dealers have a comparative advantage in managing their security portfolio. Other market participants only realize a return of γR from these assets with $\gamma < 1$. The exact assumption on γ is given below.

Dealers have cash of their own, but can also use a repo transaction to borrow the endowment of young investors for investment in securities, which is the collateral backing the repo. The repo market is imperfectly competitive

[8]As discussed in the introduction, a more general model could assume that all market participants have access to the long-term technology and then show that only a small subset of them would become dealers in order to avoid the duplication of fixed costs. We simplify the analysis by assuming that only dealers can operate the long-term technology.

[9]To our knowledge, the need to assume such capacity constraints (or more generally, decreasing returns) in dynamic models of liquidity provision has first been pointed out by van Bommel (2006).

in the sense that dealer entry into the market for borrowing in repo is possible at a small one-time cost $\phi > 0$. We assume that the total investment capacity $M\overline{I}$ strictly exceeds the investors' amount of cash available for investment in the repo market, N:

$$\overline{I} > \frac{N}{M}. \tag{2}$$

A model in which this inequality did not hold would be less realistic or analytically interesting. There would be no competition among dealers for borrowing cash from investors in the repo market. Dealers could extract all the surplus from investors by simply offering to repay the storage return of one each period, and there would be no instabilities or runs. To simplify notation, we assume from now on that $M = N$ and therefore $\overline{I} > 1$.

If a dealer in period t invests I_t, borrows b_t from young investors, promises a repurchase price (gross interest) r_{1t} for repurchase after one period and r_{2t} for repurchase after two periods, impatient investors do not roll over their loans when middle-aged, and patient investors do not roll over their loans when old, then his expected cash flow at date τ is

$$\pi_\tau = RI_{\tau-2} + b_\tau - \alpha r_{1\tau-1} b_{\tau-1} - (1-\alpha) r_{2\tau-2} b_{\tau-2} - I_\tau - c \tag{3}$$

The dealer's objective at each time t then is to maximize the sum of discounted expected cash flows $\sum_{\tau=t}^{\infty} \beta^\tau \pi_\tau$, where $\beta < 1$. In order to make the problem interesting, we assume that dealers are sufficiently patient and long-term investment is sufficiently profitable:

$$\beta^2 R > 1. \tag{4}$$

We further assume that dealers cannot borrow over and above their borrowing in the repo market, i.e. that $\pi_t \geq 0$ at all times. If dealers could borrow freely, liquidity shortages and runs would not occur, and an analysis of the repo market would be pointless.

We can now state our precise assumption on the return of assets that are transferred to other investors:

$$\beta\gamma R < 1 \tag{5}$$

We do not model the reasons for the discount γ, which in practice can be numerous. For example, as discussed in the next section, assets used for hedging purposes and arbitrage operations can typically only be mobilized at a loss. γ can also express an uncertainty discount reflecting asymmetric information between dealers and other market participants. Assumption (5) is an important element of our theory of illiquidity. It means that dealers and investors value the existing assets of other dealers strictly less than cash, and thus implies that investors have an incentive to participate in a run if one occurs.

3.2 Interpretation

The environment we have described is similar to the dynamic banking model of Qi (1994), however, the interpretation is different. The standard theory is of investors who hold deposit contracts with commercial banks, which in turn hold loans that are typically not marketable, opaque, and not pledgeable. In our model, we consider securities dealers who finance their securities with repos. Investors make one-period (overnight) loans to dealers that are collateralized by the securities (the long-term technology). Investors have the full claim on their individual collateral if the dealer cannot pay the promised interest rate. To the extent that dealers' repo transactions are part of a "shadow banking system," a banking model seems to provide an adequate formalization.

Although the securities are marketable and pledgeable, they are less liquid than cash and give a higher return to dealers than to outside investors. We interpret this higher return as being generated in several possible ways that are outside of the model but that correspond with the standard functions of securities dealers in practice. Each of these functions reduces the liquidity of the security and requires the expertise and other resources of the dealer. A dealer can use the security to act as an intermediary to other leveraged financial firms, to act as a market maker, to hedge other securities, to do risk management on a broad portfolio, to arbitrage other securities, and to conduct outright speculation.

As an example, consider the dealer as an intermediary who rehypothecates an off-the-run Treasury bond and acts as a prime broker to finance a hedge fund. At date t, the dealer borrows money from the investors and lends the money to the hedge fund. The hedge fund buys the off-the-run Treasury bond, which serves as collateral to the dealer, who then passes it on as collateral to the investors. The hedge fund can arbitrage the off-the-run Treasury bond against a separately held on-the-run Treasury bond, provided the hedge fund can keep the arbitrage on until date $t + 2$, when the spread between the Treasuries comes closer into alignment and the hedge fund can unwind the trade for a profit. A positive return is generated based on the collateral value of the Treasury bond, the hedge fund's ability to identify the arbitrage and predict how long it will take to converge, the liquidity and price efficiency provided to the Treasury market from the arbitrage, and the dealer's value as a financial intermediary. If instead, investors do not roll over the repo at date $t+1$, then the dealer does not roll over the loan to the hedge fund. The loans cannot be repaid because the hedge fund would have to unwind the arbitrage at a loss and default on the dealer, who would have to default on the investors. Instead, the investors receive the Treasury bond. The investors have to sell the off-the-run Treasury at date $t + 1$ at a loss or hold it until date $t + 2$, taking on interest rate risk and liquidity risk. The investors know less than the dealer and the hedge fund about the interest rate and liquidity risk of the Treasury, do not benefit from collateral value of the bond, and receives a lower return from the bond than the hedge fund and dealer would receive.

Next, consider an example of the dealer as a market maker, who profits by providing liquidity to the market. The investors lend at date t to the dealer, who buys an illiquid private label mortgage-backed security (MBS) and uses it as inventory to make a market in the security. The dealer makes a return of R through his bid-ask spread by buying and selling the MBS over two periods. However, the dealer needs to have the full two periods to ensure that he has time to deal in the security to make the bid-ask spread and sell his inventory at the ask price. If the investors do not roll over the repo, the dealer cannot repay because he would have to liquidate his position at a firesale loss. The investors receive the collateral, but cannot extract the full value from the security as a market maker and instead lose value by selling the bond in an illiquid market.

4 Steady-state without runs

As a benchmark, we consider symmetric steady-state allocations in which in each period young investors lend their cash to dealers and do not roll over their loan at the time of their liquidity shocks. Hence, in every period, each dealer obtains loans from a mass $N/M = 1$ of young investors, and repays middle-aged investors who do not roll over their loan and old investors. We assume that the Law of Large Numbers also holds at the level of the dealer: each period each dealer gets a representative sample of young investors. Therefore, each dealer's realized cash flow is equal to his expected cash flow (3).

A steady state then is a vector (b, I, r_1, r_2), where $b \leq 1$ is repo borrowing, $I \leq \overline{I}$ investment, r_1 repayment for one-period borrowing, and r_2 repayment for two-period borrowing, all per dealer.

Lemma 1: $r_2 = r_1^2$.

Proof: Clearly, $r_2 \geq r_1^2$, because otherwise investors would strictly prefer to never roll over their loans, regardless of their type. Patient middle-aged investors would withdraw their funds and then invest again with young investors. Suppose that this inequality is strict. In this case, an impatient middle-aged investor will optimally roll over the loan (and keep his collateral) and at the same time borrow the amount $r_1+\varepsilon$ on the market at interest rate r_1-1 (note that this loan can be collateralized). He can then claim back r_2 from the dealer one period later and repay his one-period loan $(r_1+\varepsilon)r_1$ which is feasible and profitable if $\varepsilon > 0$ is sufficiently small.

The proof is based on a simple no-arbitrage argument. It is different from the classical argument made by Jacklin (1987) in the context of the Diamond-Dybvig model, because investors in our context do not have access to the long-term investment technology. It is also different from the argument by Qi (1994), who assumes strict concavity of the investors' utility. In our market context, the no-arbitrage argument is natural and sufficient.

Hence, we can describe steady states by a triple (b, I, r), where $r = r_1$. The steady-state budget identity of individual dealers is

$$RI + b = I + \alpha rb + (1-\alpha)r^2 b + c + \pi \tag{6}$$

where the left-hand side are the total inflows per period and the right-hand side total outflows. Clearly, if $R > 1$, the higher I the better. We do not concern ourselves with showing how a steady state with $I > 1$ could emerge if there were a startup period. But under our assumption (4) that dealers are sufficiently patient, it is clear that dealers have an interest in building up maximum investment from lower investment levels.[10]

Clearly, (b, I, r) must be such that $\pi \geq 0$, because otherwise dealers would leave the market. Steady states with no borrowing are uninteresting in our context. We now characterize the steady states with $b > 0$ by a sequence of simple observations.

Lemma 2: *If $r > 1$, steady-state repo borrowing is maximal: $b = 1$.*

Proof: The supply of loanable funds per dealer is inelastically equal to 1 if $r > 1$. Hence, the lemma follows from market clearing.

Lemma 3: *Steady-state investment is maximal: $I = \overline{I}$.*

Proof: Suppose the Lemma is wrong. An individual dealer can then increase investment slightly at any date t by using his own cash. By condition (4), this yields a strict increase in discounted profits.

Lemma 4: *Steady-state repo rates satisfy*

$$(1-\alpha)\beta^2 r^2 + \alpha\beta r = 1 \tag{7}$$

Proof: For each unit of cash that the dealer borrows and invests at date t, he pays back αr in $t+1$, generates returns R in $t+2$ and pays back $(1-\alpha)r^2$ in $t+2$. Hence, his expected discounted profits on this one unit is $\beta^2(R - (1-\alpha)r^2) - \beta\alpha r$. Alternatively he could invest his own cash. The discounted profits from not borrowing the one unit and rather investing his

[10]The literature has not always been clear about the distinction between investment capacity ($\overline{I}$ in our model) and per capita borrowing (1 in most models). In particular, the implicit assumption that $\overline{I} = 1$ in Qi (1994), Bhattacharya and Padilla (1996) and Fulghieri and Rovelli (1998) is not necessary, and may even ignore interesting dynamic features. See van Bommel (2006) for an excellent discussion.

own money is $\beta^2 R - 1$. In steady state this cannot be strictly better, which implies $(1-\alpha)\beta^2 r^2 + \alpha\beta r \leq 1$.

Suppose that this inequality is strict. As argued above, this means that investment through borrowing is strictly preferred to investing the dealer's own cash. But since $\overline{I} > 1 \geq b$, by Lemma 3 in steady state dealers use some of their own cash to invest. Hence, an individual dealer can increase his repo rate r slightly, without violating the strict inequality, and thus attract additional cash from investors. Because the inequality was assumed to be strict, this makes him strictly better off.

We call condition (7) the dealer's "repo participation constraint". Let $\overline{r}$ be the solution to (7). Basic algebra shows that $\overline{r} = 1/\beta > 1$. This makes sense: at the margin, dealers discount profits with the repo market interest rate. But it is interesting to note that $\overline{r}$ does not depend on the supply and demand characteristics R and α. Furthermore, as we shall discuss now and differently from standard models of financial price competition, dealers make positive profits at this interest rate, if the fixed costs c are not too high.

From (3), the dealer's expected steady-state profits with $b = 1$ and $r = 1/\beta$ are

$$\pi = (R-1)\overline{I} + 1 - \frac{\alpha}{\beta} - \frac{1-\alpha}{\beta^2} - c. \tag{8}$$

To make the analysis interesting we will assume that these profits are positive, i.e. that R, $\overline{I}$, α, or β are sufficiently large or c is sufficiently low. All of these assumptions are reasonable and consistent with our previous assumptions.

Assumption: Period costs satisfy

$$c < (R-1)\overline{I} + 1 - \frac{\alpha}{\beta} - \frac{1-\alpha}{\beta^2} \tag{9}$$

Note that this assumption concerns parameters, not equilibrium values. An important and novel feature of our model therefore is that condition (7) prevents competition from driving up repo rates to levels at which dealers make zero profits. The reason why repo profits are positive is intuitive (but not trivial): dealers must have an incentive to use their investment opportunities on behalf of investors instead of using internal funds to reap those profits for themselves. This rationale of positive intermediation profits is different from the traditional banking argument of positive franchise values (e.g., Bhattacharya, Boot, and Thakor (1998), or Hellmann, Murdock and Stiglitz, (2000)), as it explicitly recognizes the difference between internal and external funds.

Proposition 1 *Assume that (9) holds. Then the model has exactly one steady state in which investors roll over repo contracts according to their liquidity needs. In steady state,* $b = 1, I = \overline{I}, r = \overline{r}, \pi > 0$.

Proof: By Lemmas 2-4, if there is a steady state it is of the form given in the proposition. Conversely, these choices are optimal for young investors, and, since $r > 1$, patient middle-aged investors find it optimal to roll over their repos.[11] By (9), dealers make positive profits and therefore prefer steady-state borrowing to autarky.

Proposition 1 shows that the model has a unique no-run steady state and that dealers make strictly positive profits, if periodic fixed costs are not too high (Assumption (9)). Note that this steady state is robust to competition from new dealers for any small cost $\phi > 0$ to enter the repo market. The reason is that there are no gains from repurchase transactions over and above what can be gained by private investment because the repo participation constraint binds. Hence, outside banks with access to the long-term investment technology have no incentive to enter the repo market.

[11] In typical repo contracts, investors do not have the right to keep the collateral instead of accepting the repurchase payment. This is what we assume here. Hence, the choice of an investor is between accepting the repayment and storing it (yielding r next period) or rolling the contract over (yielding r^2 next period). If the investor has the right to refuse the repayment and keep the security instead, assumption (5) still implies that patient middle-aged investors find it optimal to roll over their repos.

5 Repo runs without asset sales

In this section, we study the stability of dealers in the face of possible runs. We analyze this problem under the assumption that behavior until date t is as in Proposition 1 and ask whether the beliefs that all investors of a given dealer will refuse to roll over their loans at date t can be self-fulfilling. The interaction between dealer and investors at a given date t is as follows. First, the dealer offers new investors repo contracts specifying that each investor lends the dealer 1 and gets a newly invested security as collateral that will be returned to the dealer next period if the investor is repaid r. Simultaneously, the dealer offers middle-aged investors that they keep their collateral for one more period and then return it against a repayment of r^2. Then, new and middle-aged investors simultaneously announce whether they lend or roll over their repo loan, respectively. Finally, if the dealer can satisfy all repayment demands in cash, she does so and invests the amount I_t.

If instead the dealer cannot satisfy all withdrawals in cash, then she defaults. In this case, the young investors get their 1 unit of funds back. Middle-aged investors who have demanded repayment receive all the cash available pro-rata, and in addition keep their collateral up to the amount of their loan. In contrast, investors who had agreed to roll over their loan simply keep their collateral.

We examine symmetric subgame-perfect Nash equilibria of the above game between investors and the given dealer. The key question is how much cash the dealer can mobilize to meet the repayment demands by middle-aged investors. At the beginning of the period, a dealer, on the asset side of his balance sheet, holds $R\overline{I}$ units of cash from investments at date $t-2$, as well as securities that will yield $R\overline{I}$ units of cash at date $t+1$. The dealer holds maturing repos on the liability side of its balance sheet. As derived in the preceding section, if young investors provide fresh funds, the dealer has enough cash to repay the loans of (old) patient investors born in $t-2$ and (middle-aged) impatient investors born in $t-1$ who will not roll over for sure. Depending on the size of current cash flows and on whether young investors provide fresh funds, the dealer may not hold enough cash to also repay patient investors born in $t-1$ if they choose not to roll over their loans.

Given the assumption about the treatment of investors in bankruptcy, it is a weakly dominant strategy for young investors to provide fresh funds.[12] We will therefore assume that young investors indeed always provide fresh funds. Then, the run demand can be satisfied by the individual dealer if $\pi \geq (1-\alpha)r$, which is equivalent to

$$(R-1)\overline{I} \geq r + (1-\alpha)r^2 - 1 - c \tag{10}$$

But more is possible. In the event of a run at date t, the cash position of the individual dealer who satisfies the run demand is

$$I_0 = R\overline{I} + 1 - r - (1-\alpha)r^2 - c \tag{11}$$

Clearly, if $I_0 < 0$ the dealer does not have the liquidity to stave off the run, and the dealer is bankrupt. However, if $I_0 \geq 0$, but (10) does not hold, the dealer can invest less than the steady state level $\overline{I}$ in order to liberate cash to accommodate the run demand. This yields a lower return in $t+2$, but it is still consistent with continuing borrowing the full amount of 1 from investors and making the full steady state repayments in the future, because the dealer can make payments out of his date $t+2$ profits to cover the shortfall resulting from (a limited degree of) underinvestment. In the limit, the dealer can exhaust all of his profits at date $t+2$ and reduce investment in t correspondingly by π/R. In fact, he can carry this further. At date $t+2$ he can reduce investment below the steady state level to liberate cash that can be used to meet the shortfall resulting from a further reduction in investment at date t, etc. This way, the dealer can reduce investment in future periods $t+2k$, $k = 1, 2, ...$, in order to shift profits forward to date t, which allows him to liberate more and more of the current cash to accommodate the run demand.

Lemma 5: *In response to a run, the optimal sequence of investments at dates $t+2k$, $k = 0, 1, 2, ...$, reduces profits to zero up to a certain period*

[12] Other assumptions about the treatment of investors in bankruptcy yield even tighter constraints for dealer solvency and liquidity, because then young investors may not provide liquidity in case of a run.

(which can be ∞), from which on investment is back to the steady state level $\overline{I}$.

Proof: Suppose that there is a period $t+2k_0$ in which investment is smaller than $\overline{I}$ and the dealer makes positive profits. Then the dealer can reduce profits in $t+2k_0$ slightly by investing $\delta > 0$ more, which yields $R\delta$ in $t+2k_0+2$. From then on he sticks to the former investment sequence. By (4), he is strictly better off. To complete the proof note that if there is a period in which the dealer can invest $\overline{I}$, then he can do so ever after and this is optimal.

The policy identified in Lemma 5 is a value reducing distortion of investment. But since it allows the dealer to generate current cash and keep the business alive, it is preferred to going bankrupt. The amount of cash the dealer can free up at date t by following this strategy is

$$\pi + \frac{1}{R}\pi + \frac{1}{R^2}\pi + ... = \frac{R}{R-1}\pi \tag{12}$$

But in fact, more is possible. The above investment strategy does not involve the dealer's behavior at dates $t+2k+1$, $k = 0, 1, 2, ...$ If the dealer sets an amount S aside out of date $t+1$ - profits and stores it until $t+2$, then he can reduce investment in t by S/R, by using S in $t+2$ to cover the shortfall. By the same logic as above, the dealer can now increase the amount set aside in $t+1$ by reducing investment in $t+1$ and making up for the shortfall in $t+3$ by using profits from $t+3$, etc. As in Lemma 5, it is straightforward to show that the optimal strategy in periods $t+2k+1$ features maximum investment for as long as possible. By following this strategy, the dealer can again bring all future profits π from periods $t+2k+1$ forward. Moreover, after a run, in period $t+1$ there is more cash than just π because the amount $(1-\alpha)r^2$ is no longer due, having been withdrawn already in the run. Hence, the total amount of cash that can be freed up at date t by bringing profits from periods $t+2k+1$ forward is

$$\frac{1}{R}(\pi + (1-\alpha)r^2) + \frac{1}{R^2}\pi + ... = \frac{1}{R}(1-\alpha)r^2 + \frac{1}{R-1}\pi \tag{13}$$

(12) and (13) together yield the total amount of cash that the dealer can

potentially liberate at date t if he has the liquidity:

$$\begin{aligned} F_0 &= \frac{R}{R-1}\pi + \frac{1}{R}(1-\alpha)r^2 + \frac{1}{R-1}\pi \\ &= \frac{1}{R}(1-\alpha)r^2 + \frac{R+1}{R-1}\left((R-1)\overline{I} + 1 - \alpha r - (1-\alpha)r^2 - c\right) \end{aligned}$$

where we have inserted π from (8). Whether or not the dealer can indeed pay out F_0 depends on his liquidity, $\max(0, I_0)$. Hence, the actual amount of cash the dealer can liberate in response to the run is $\min(max(0, I_0), F_0)$. The dealer can therefore avert the run if the following liquidity and solvency constraints are both satisfied:

$$I_0 \geq 0 \tag{14}$$

$$F_0 \geq (1-\alpha)r \tag{15}$$

Condition (14) requires that the dealer has the cash needed to satisfy the additional liquidity demand in the run. Condition (15) makes sure that the dealer can mobilize enough future returns by adjusting his investment behavior. The former constraint refers to the dealer's (current) liquidity, the latter to his overall solvency.

Using the steady-state value $r = 1/\beta$ in the above two conditions and re-arranging, we get the following result, where we re-introduce the explicit reference to the dealer in question.

Proposition 2 *In steady state, a run on dealer m is impossible if the dealer's liquidity and solvency constraints hold, i.e. if*

$$\beta^2 R\overline{I} \geq 1 - \alpha + \beta - (1-c_m)\beta^2 \tag{16}$$

$$\text{and } \beta^2 R\overline{I} \geq \left(\frac{R}{R-1} + \frac{\beta R - 1}{R+1}\right)(1-\alpha) + \frac{\alpha\beta R}{R-1} - \frac{R\beta^2}{R-1}(1-c_m) \tag{17}$$

If any of these two conditions is violated and the dealer cannot sell his assets, then a run cannot be prevented if it occurs and the run bankrupts the dealer.

Proof: Suppose conditions (16) and (17) hold and all patient middle-aged investors demand repayment. Since the young investors provide fresh funds, the preceding argument has shown that the dealer can satisfy the run demand. Middle-aged patient investors therefore receive r, which they can store until $t+1$. If such an investor does not run, he receives r^2 in $t+1$. He therefore strictly prefers not to run.

If any of the two conditions (16) or (17) is violated, the dealer cannot satisfy the run demand. Each middle-aged patient investor therefore receives a payment in cash of ωr, where $\omega \in (0,1)$ is the fraction of the return due, plus some of his collateral. If $\gamma R < r(1-\omega)$ he keeps all of his collateral and thus has a total payoff of $\omega r + \gamma R$ in $t+1$. If $\gamma R \geq r(1-\omega)$ he keeps the fraction $r(1-\omega)/\gamma R$ of the collateral and thus has a total payoff of r in $t+1$. If, alternatively, he does not run he gets his collateral value γR. By (5) he strictly prefers to run in both cases.

Note that either of the conditions in Proposition 2 may be binding. In fact, (16) binds for $\alpha = 0$ and β not too small, while (17) binds for $\alpha = 1$. Furthermore, the higher the period costs c_m, the tighter are both constraints. This is intuitive, because higher c mean lower profits, hence smaller buffers against the run. Note that even in the extreme case in which dealer activity is costless ($c_m = 0$) (16) and (17) may not hold. In the relevant case of intermediate costs, the inequalities can go both ways. If c_m approaches the bound in (9) the inequalities are obviously reversed, because per period profits become so small that neither current nor future profits are enough to stave off the run.

Proposition 2 shows that if dealers have sufficient access to profitable investment ($\overline{I}$ sufficiently large) or if these investment opportunities are sufficiently profitable (R sufficiently large), dealers can stave off runs individually, only by reducing their investment temporarily and shifting profits forward in time. In this case, runs cannot occur, even out of steady state. If one of the two conditions in Proposition 2 is violated, i.e. if the dealer is expected to be illiquid or insolvent in case of a run, a run would bankrupt the individual dealer if the dealer cannot sell his illiquid assets. A run would therefore be a self-fulfilling prophecy and can upset the steady state.

6 Runs and Asset Sales

In this section, we introduce the possibility of asset sales as a reaction to a run. As in the last section, we first consider a situation where the investors of only one dealer may run. In the next section, we consider the case where there is a potential run on the whole market. We ask the same question as in the last section: if behavior until date t is steady state as in Proposition 1, can the beliefs that all investors of a given dealer will refuse to roll over their loans at date t be self-fulfilling?

At date t, the dealer, indexed by, say, m, holds assets that will yield $R\bar{I}$ at date $t+1$, but nothing at date t. We assume that in response to the run, the dealer can sell his illiquid assets to other dealers at some market price p. The price other dealers are ready to pay will depend on the value they can realize from these assets in the future and on their own cash available for asset purchases.

If the dealer under distress sells an amount A of assets, this improves his current liquidity by pA and worsens his solvency through a reduction of $(t+1)$-cash by RA. Since $p \leq \beta R$, this immediately implies that asset sales are of no help if the dealer's solvency condition is violated:

Lemma 6: If the dealer's solvency condition (17) does not hold, then asset sales cannot stave off a run.

If, on the other hand, the solvency condition is slack, then the dealer can use asset sales in order to improve his current liquidity. If his liquidity constraint (16) holds as well, then a run does not occur, as seen in the last section. Off the equilibrium path, the dealer then can trade off the costs and benefits of selling assets against the losses from bringing forward future profits without trading. If his liquidity constraint is violated, he is forced to sell assets.

Remember from (11) in the last section that his liquidity constraint is violated if $I_0 < 0$. To facilitate notation denote the dealer's net steady state repayments to investors (including fixed costs) by

$$n_m = \alpha r + (1-\alpha)r^2 - 1 + c_m \tag{18}$$

From now on, we suppress the index m for ease of notation. If the investor sells A of his current long-term assets, satisfies the run demand, saves the amount S of cash at date $t+1$ to date $t+2$ in order to reduce investment in t by S/R, and forgoes profits during the first T periods from date t on until the investment level reaches $\overline{I}$ ($T \leq \infty$), then the sequence of investments, starting at date t, is

$$
\begin{aligned}
I_0 &= R\overline{I} + 1 + pA - r - (1-\alpha)r^2 - c = R\overline{I} + pA - n - (1-\alpha)r && (19)\\
I_1 &= R(\overline{I} - A) + 1 - \alpha r - S - c = R(\overline{I} - A) - n + (1-\alpha)r^2 - S && (20)\\
I_2 &= RI_0 + 1 + S - \alpha r - (1-\alpha)r^2 - c = RI_0 - n + S && (21)\\
I_3 &= RI_1 + 1 - \alpha r - (1-\alpha)r^2 - c = RI_1 - n && (22)\\
I_4 &= RI_2 + 1 - \alpha r - (1-\alpha)r^2 - c = RI_2 - n && (23)\\
&\quad \text{etc.} && (24)
\end{aligned}
$$

Writing out the two recursions in equations (19) - (24) and re-arranging yields

$$
\begin{aligned}
I_{2k} &= R^k I_0 - \frac{R^k - 1}{R-1} n + R^{k-1} S && (25)\\
&= R^{k-1}\left[RI_0 - \frac{R}{R-1} n + S\right] + \frac{n}{R-1} && (26)\\
I_{2k+1} &= R^k I_1 - \frac{R^k - 1}{R-1} n = R^k \left[I_1 - \frac{n}{R-1}\right] + \frac{n}{R-1} && (27)
\end{aligned}
$$

for $k \geq 1$. Each of these two recursions comes to an end when $I_n \geq \overline{I}$. Equations (25) and (27) directly imply

Lemma 7: The sequences I_{2k} and I_{2k+1} satisfy

$$
\begin{aligned}
I_{2k} &\nearrow \infty \Leftrightarrow RI_0 > \frac{Rn}{R-1} - S && (28)\\
I_{2k} &\searrow -\infty \Leftrightarrow RI_0 < \frac{Rn}{R-1} - S && (29)\\
I_{2k+1} &\nearrow \infty \Leftrightarrow I_1 > \frac{n}{R-1} && (30)\\
I_{2k+1} &\searrow -\infty \Leftrightarrow I_1 < \frac{n}{R-1} && (31)
\end{aligned}
$$

Because profits cannot be negative, the proposed investment strategy is infeasible if (29) or (31) hold. Conversely, if and only if (28) and (30) hold with weak inequality, $S \geq 0$, and the initial feasibility constraints $I_0 \geq 0$ and $I_1 \geq 0$ hold, then the proposed strategy is feasible. Note that (30) implies $I_1 \geq 0$. Using (19) and (20), we can express (28) and (30), as well the remaining feasibility constraint, in terms of the steady-state investment level $\overline{I}$:

$$\begin{aligned} I_0 &\geq 0 \Leftrightarrow R\overline{I} \geq n + (1-\alpha)r - pA && (32)\\ I_{2k} &\nearrow \Leftrightarrow R\overline{I} \geq \frac{R}{R-1}n + (1-\alpha)r - pA - \frac{1}{R}S && (33)\\ I_{2k+1} &\nearrow \Leftrightarrow R\overline{I} \geq \frac{R}{R-1}n - (1-\alpha)r^2 + S + RA && (34) \end{aligned}$$

Conditions (33) and (34) are only compatible if

$$\begin{aligned} \overline{I} - \frac{1}{R-1}n + \frac{1-\alpha}{R}r^2 - A &\geq -R\overline{I} + \frac{R}{R-1}n + (1-\alpha)r - pA \\ \Leftrightarrow R\overline{I} &\geq \frac{R}{R-1}n + \frac{R}{R+1}(1-p)A + (1-\alpha)\frac{r(R-r)}{R+1} \\ \Leftrightarrow (p-1)A &\geq \frac{R+1}{R-1}n + (1-\alpha)\frac{r(R-r)}{R} - (R+1)\overline{I} \quad (35) \end{aligned}$$

Conversely, if (35) holds, then it is possible to find an S such that (33) and (34) hold. Yet, it is also necessary that $S \geq 0$. From (34), this can be achieved if and only if:

$$A \leq \overline{I} - \frac{n}{R-1} + (1-\alpha)\frac{r^2}{R} \quad (36)$$

If (36) is violated, then the dealer's asset position at date $t+1$ is so small that he will eventually become insolvent (I_{2k+1} will become negative).[13]

Hence, (32), (36), and (35) are necessary and sufficient for asset sales to be able to prevent a run. We now re-introduce the reference to c_m in order to make the dependence of these conditions on the dealer's cost structure explicit. The argument above implies that whether or not a run can be prevented depends on the dealer's costs and the market value of his assets in case of a fire sale:

[13] Note that the right-hand side of (36) is strictly positive by (9).

Proposition 3 *A run on dealer m cannot succeed iff the solvency constraint (17) holds and the market price of the dealer's long-term assets satisfies*

$$\begin{aligned} p &\geq \overline{p} \qquad (37) \\ \textit{where } \overline{p} &= R(R-1)\frac{1-\alpha+\beta-\beta^2(1-c_m)-\beta^2 R\overline{I}}{\beta^2 R(R-1)\overline{I}+\beta^2 R(1-c_m)-\alpha\beta R-1+\alpha} < 1 \end{aligned}$$

Otherwise a run cannot be prevented if it occurs and bankrupts the dealer.

Proof: If the solvency constraint does not hold, a run is successful if it occurs by Lemma 6. So assume that the solvency constraint holds.

There is an A such that conditions (36) and (32) are compatible iff

$$p\left(\overline{I}-\frac{n}{R-1}+(1-\alpha)\frac{r^2}{R}\right) \geq n+(1-\alpha)r-R\overline{I} \qquad (38)$$

The left-hand side of (38) is strictly positive by (9). If the liquidity constraint (16) holds, the right-hand side of (38) is not positive (hence, (38) holds trivially) and runs can be staved off by Proposition 2. Suppose the liquidity constraint does not hold. Then dividing through by the round bracket on the left hand side and inserting $r = 1/\beta$ yields (37). The solvency constraint implies that $\overline{p} < 1$.

The right-hand side of (35) is negative because the solvency constraint holds. Therefore, (35) and (32) are trivially compatible if $p \geq 1$. It also can be easily verified that the two are compatible if $p = \overline{p}$. Hence they are compatible for all $p \geq \overline{p}$.

On the demand side for assets, the cash available to buy up the distressed dealer's assets is given by a similar consideration as in the previous section. Each of the $M-1$ healthy dealers has π_i in terms of current profits and can bring forward profits of π_i of each future period by adjusting his investment policy. As argued above, the total sum of profits that an individual dealer can potentially mobilize in period t therefore is

$$F_i = \pi_i + 2\sum_{\tau=1}^{\infty}\frac{1}{R^\tau}\pi_i = \pi_i + \frac{2}{R-1}\pi_i = \pi_i + f_i = \frac{R+1}{R-1}\pi_i$$

Furthermore, the buyers of the distressed assets receive γR per unit of the asset in $t+1$ (where $0 \leq \gamma < 1$), which they can store until $t+2$, and thus further reduce investment in t by $\gamma R/R$. At date t, each of the $M-1$ healthy dealers has

$$I_i = R\overline{I} + 1 - \alpha r - (1-\alpha)r^2 - c_i$$

in cash, after having received funds from new investors and repaid maturing repos to existing investors, but before investing in new long-term assets.[14]

If individual healthy dealers each purchase A_i of the distressed assets ($\sum_{i=1}^{M-1} A_i = A$), each must invest at least $\overline{I} - (f_i + \gamma A_i)$ in new long-term assets. Hence, his cash available at date t is

$$I_i - \left(\overline{I} - (f_i + \gamma A_i)\right) = F_i + \gamma A_i$$

Total cash in the market therefore is

$$C = \frac{R+1}{R-1} \sum_{i \neq m} \pi_i + \gamma A. \tag{39}$$

We can now characterize the demand for assets by healthy dealers. It is straightforward to see that he highest price healthy dealers will pay for securities is the price at which the discounted return of the securities is equal to the opportunity cost of the cash needed to purchase the securities.

Lemma 8: If $p > \beta\gamma R$, healthy dealers do not buy any of the distressed dealer's assets. If $p < \beta\gamma R$, demand for the assets is at least $\left(\sum_{i \neq m} \pi_i\right)/p$. and is decreasing in p.

While a full characterization of the demand for assets is of little interest, an important special case deserves special attention:

Proposition 4 *If the solvency constraint of dealer m is violated or if his liquidity constraint is violated and $\beta\gamma R < \overline{p}$, then a run on dealer m cannot be staved off and bankrupts the dealer if it occurs.*

[14] Note that $I_i = \pi_i + \overline{I} > 0$.

The proof follows directly from Proposition 3. As seen in the proposition, if the dealer's solvency constraint is violated, asset sales cannot help to raise the cash necessary to stave off the run if it occurs. If the solvency constraint holds, but the liquidity constraint is violated, then (37) shows that the distressed dealer can only survive if he can sell his assets at a price of at least $\overline{p}$. However, Lemma 8 shows that for any such price, the demand for these assets is zero. Hence, the market for distressed assets cannot operate, and the run, if it occurs, bankrupts the dealer.

7 Market Runs

As noted above, the more dealers are in trouble, the more assets troubled dealers are trying to sell and the fewer dealers are available to buy these assets. This puts pressure on the price of assets and it makes it less likely that a run can be avoided. In the extreme case of a market run, no dealer is available to buy assets, and dealers are in the same situation as if their assets were not marketable. Hence, the conditions of Proposition 2 are the relevant ones in order to evaluate the possibility of runs.

As Proposition 2 shows, the possibility of a run against dealer m depends on his costs c_m. We therefore get the following classification of dealers in the case of a market run.

Proposition 5 *There is a critical threshold $\overline{c} \geq 0$ such that in the case of a market run all dealers with $c_m \leq \overline{c}$ are able to stave off the run and all dealers with $c_m > \overline{c}$ are bankrupted.*

The critical threshold $\overline{c}$ is the largest value c_m such that conditions (16) and (17) in Proposition 2 both hold. While our theory is a theory of multiple equilibria and therefore cannot predict runs, Proposition 5 makes a precise prediction about the outcome of a market run if one is attempted: The weakest firms in terms of their cost structure must fail, while the stronger ones cannot fail.

8 Extension: Liquidity provision

Access to a lender of last resort is a standard tool used to strengthen the banking sector in the face of financial fragility. Theoretical work has shown how access to a lender of last resort can prevent bank runs (see, for example, Allen and Gale 1998, Martin 2006, Skeie 2004). In the U.S., the broker dealers that rely on the tri-party repo market as a source of short-term funding did not have direct access to discount window. This lack of access to emergency liquidity proved destabilizing during the crisis and motivated the Federal Reserve to introduce the Primary Dealer Credit Facility (PDCF). Similar concerns about money market mutual funds, who represent an important share of investors in the tri-party repo market, motivated the creation of the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF), and the Money Market Investor Funding Facility (MMIFF). These facilities were created under section 13.3 of the Federal Reserve Act, which allows the Federal Reserve to lend to a variety of institutions under unusual and exigent circumstances. As such, these facilities are temporary.[15]

The Task Force on Tri-Party Repo Infrastructure (2009) notes the need to "Consider establishing an industry-sponsored utility with the ability to finance the securities portfolio of a faltering or defaulted dealer and limit the associated stress on the market while their portfolio is liquidated." The model in our paper suggests that there would be benefits to the creation of a lender-of-last-resort facility for the tri-party repo market. The argument is similar to the case of banking. In case of a run, investors do not refuse to roll over their loans because they need cash, but because they are concerned about the default of the dealer and having to hold collateral that they might have to liquidate. As in Allen and Gale (1998), Martin (2006), or Skeie (2004), a lender of last resort could lend cash to the dealer taking securities as collateral. The cash could be used to pay all investors who do not roll over their loans. This would prevent the default of the dealer and allow it to manage the collateral until it matures. Knowing that the dealer will not default, investors no longer have to worry about having to hold or liquidate

[15] The MMIFF expired on October 30, 2009. The Board of Governors approved extension of the AMLF and the PDCF through February 1, 2010.

assets, so their incentive to run is reduced.

9 Conclusion

In this paper we study a model of short-term collateralized borrowing and the conditions under which runs can occur. Our framework resembles the dynamic model of banks studied in Qi (1994), but expands that model in a number of directions. We derive a dynamic participation constraint that must hold for dealers to agree to purchase securities on behalf of investors. Under this constraint, dealers will make profits that can be mobilized to forestall runs.

A key difference between traditional banks and modern financial intermediaries is that the former mainly hold opaque assets while the latter's assets are much more liquid and marketable. We study the role of market able assets in preventing bank runs. Without asset sales, runs can be forestalled by mobilizing current and future assets. This gives rise to two constraints that can be interpreted as a solvency and a liquidity constraint. The solvency constraint assures that there are enough current and future profits to repay all investors who do not renew their loans. The liquidity constraint guarantees that the necessary resources are available at the date the run occurs. A run can be prevented if neither constraints are violated.

Next we consider the case where dealers can sell their assets. We show that because of cash-in-the-market pricing, the price of assets will depend on the number of dealers trying to sell assets and the opportunity cost of funds for dealers willing to buy assets. As more dealers try to sell their assets, the price of the assets they sell will decline.

Asset sales can help a solvent but illiquid dealer stave of a run, as they provide an alternative way of mobilizing future profits. However, we show that the price of the assets cannot be so high that the solvency constraint is relaxed. If the liquidity constraint binds, but the solvency constraint is slack, dealers can relax the liquidity constraint even if the price of assets is low. In the limit, however, as all dealers are affected by a run, no dealer is available to purchase assets. In this extreme case, asset sales cannot help dealers.

Our framework can be used to consider interesting policy questions related to the fragility of the tri-party repo funding mechanism. For example, Lehman's demise highlighted an important problem: There is no framework to unwind the positions of any large bank that deals in repo should it fail. Lehman required large loans from the Federal Reserve Bank of New York to settle its repo transactions (WSJ 2009). Our framework can be used to study a liquidation agent, as suggested in the Task Force on Tri-Party Repo Infrastructure (2009), that could be used to unwind the positions of a defaulting dealer.

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